COURSE DATA

Data Subject	
Code	44080
Name	Seminar on geometry and topology
Cycle	Master's degree
ECTS Credits	3.0
Academic year	2024 - 2025

Study (s)

Degree	Center	Acad. year	Period
2183 - Master's Degree in Mathematical Research	Faculty of Mathematics	1	Second term

Subject-matter

Degree	Subject-matter	Character
2183 - Master's Degree in Mathematical Research	4 - Specialty in fundamental mathematics	Optional

Coordination

Name	Department
MIQUEL MOLINA, VICENTE FELIPE	363 - Mathematics

SUMMARY

This course will be an introduction to the study of the isoperimetric problem on surfaces. While studying various methods to tackle the problem's solution, the course will delve into different aspects of mathematics used in this context: variational calculus, length and area measurement, Cartan-Hadamard surfaces, and some issues in partial differential equations. It is often said that things are learned when applied to specific problems, and this course relies on that statement, aiming to put it into practice. It will begin with a quick review (without too many technicalities) of differential geometry of surfaces, the divergence theorem, minimum principles, and perimeter definition.

The main part of the course will focus on studying the isoperimetric inequality in R^3 surfaces and Cartan-Hadamard surfaces (which, due to their planar topology, do not require the concept of abstract manifold).

The key theorems will involve determining the isoperimetric profile on some complete surfaces and consequently establishing the existence and uniqueness of isoperimetric regions.

PREVIOUS KNOWLEDGE

Relationship to other subjects of the same degree

There are no specified enrollment restrictions with other subjects of the curriculum.

Other requirements

As minimum requirements, the student should be familiar with basic concepts of Linear Algebra and functions of several variables, and should be reasonably comfortable working with them. It is beneficial (though not strictly necessary) for the student to have some elementary knowledge of the theory of curves in the plane and surfaces in R^3, as well as basic theorems of differential equations.

COMPETENCES (RD 1393/2007) // LEARNING OUTCOMES (RD 822/2021)

2183 - Master's Degree in Mathematical Research

- Students should apply acquired knowledge to solve problems in unfamiliar contexts within their field of study, including multidisciplinary scenarios.
- Students should demonstrate self-directed learning skills for continued academic growth.
- Que los estudiantes comprendan los conceptos y las demostraciones rigurosas de teoremas fundamentales de áreas transversales de las Matemáticas.
- Que los estudiantes comprendan los conceptos y las demostraciones rigurosas de teoremas fundamentales de alguna de las áreas específicas de las Matemáticas.
- Que los estudiantes sean capaces de aplicar los resultados y técnicas aprendidas para la resolución de problemas complejos de alguna de las áreas de las Matemáticas, en contextos académicos o profesionales.
- Que los estudiantes tengan capacidad para elaborar y desarrollar razonamientos lógico-matemáticos e identificar errores en razonamientos incorrectos.
- Que los estudiantes posean la capacidad para enunciar y verificar proposiciones en alguna de las áreas de las Matemáticas y para transmitir los conocimientos matemáticos adquiridos, oralmente y por escrito.
- Que los estudiantes sean capaces de comprender de manera autónoma artículos de investigación o innovación en alguna de las áreas de las Matemáticas.

LEARNING OUTCOMES (RD 1393/2007) // NO CONTENT (RD 822/2021)

- Understand what the isoperimetric profile is, the difficulties and techniques for determining it, and discover the necessity of giving a precise concept of perimeter.
- Acquire, at the same time, practical notions of Differential Geometry and PDEs, especially variational problems.

DESCRIPTION OF CONTENTS

1. Surface review

1.1 First fundamental form, surface area, and enclosed volume. Review of different types of curvature. Use of the divergence theorem.
1.2 Geodesic curvature, length, and area enclosed by a curve on a surface. Use of the divergence theorem on surfaces.
1.3 The coarea formula.
1.4 First and second variation formulas of volume.

2. Basic concepts related to the isoperimetric problem

2.1 Sets of finite perimeter
2.2 Minkowski content
2.3 Hausdorff measure
2.4 Isoperimetric profile

3. The isoperimetric problem on surfaces

3.1 The method of internal parallels.
3.2 Bandle's approximation.

4. Shortening flow of curves in the isoperimetric problem

4.1 Basic results of the shortening flow of curves
4.2 The principle of separation
4.3 Applications to the isoperimetric problem

5. Variational approach

5.1 Curves with constant geodesic curvature on surfaces of revolution
5.2 Planes and spheres of revolution with monotone Gaussian curvature
5.3 Surfaces with singularities

6. Complete surfaces with non-negative curvature

6.1 Existence of isoperimetric regions on these surfaces
6.2 Consequences for the isoperimetric profile

WORKLOAD

ACTIVITY	Hours	% To be attended
Theory classes	30,00	100
Development of group work	10,00	0
Development of individual work	10,00	0
Study and independent work	15,00	0
Preparation of evaluation activities	10,00	0
TOTAL	**75,00**	

TEACHING METHODOLOGY

Chalkboard classes and problem solving. Some classes will be given by the students after providing them with appropriate material.

EVALUATION

Continuous assessment, taking advantage of student interventions in class and oral exams that serve both as assessment and instruction.

REFERENCES

Basic

- Manuel Ritoré: Isoperimetric inequalities in Riemannian manifolds, Birkhäuser, 2023
- Catherine Bandle: Isoperimetric inequalities and applications, Pitman, 1980
- Y. D. Burago, V.A. Zalgaller: Geometric inequalities Springer, 1988
- D. Burago, Y. Burago, S. Ivanov: A Course in Metric Geometry, American Mathematical Society, Providence, RI, 2001)