

**COURSE DATA****Data Subject**

<b>Code</b>	43484
<b>Name</b>	Research in didactics of basic mathematics
<b>Cycle</b>	Master's degree
<b>ECTS Credits</b>	7.0
<b>Academic year</b>	2023 - 2024

**Study (s)**

<b>Degree</b>	<b>Center</b>	<b>Acad. year</b>	<b>Period</b>
2157 - M.D. in Research in Subject Didactics	Faculty of Teacher Training	1	First term
3112 - Specific Didactics	Doctoral School	0	First term

**Subject-matter**

<b>Degree</b>	<b>Subject-matter</b>	<b>Character</b>
2157 - M.D. in Research in Subject Didactics	8 - Research in didactics of mathematics	Optional
3112 - Specific Didactics	1 - Complementos de Formación	Optional

**Coordination**

<b>Name</b>	<b>Department</b>
DIAGO NEBOT, PASCUAL DAVID	85 - Mathematics Education
FERRANDO PALOMARES, IRENE	85 - Mathematics Education

**SUMMARY**

This subject aims to study in depth the main lines of research that are being developed in the didactics of mathematics and to apply the general theoretical frameworks studied in another subject in order to characterise the research in this area. The content of this subject seeks to complete specialised training in research in each subject, which will be studied in depth through the study of specific research focused on the problems of teaching and learning elementary mathematics.

To achieve the objectives and competencies that this subject must provide and to place students in a position to develop and complete research project for their master's degree final project.



## PREVIOUS KNOWLEDGE

### Relationship to other subjects of the same degree

There are no specified enrollment restrictions with other subjects of the curriculum.

### Other requirements

For the proper development of this subject, students will have to use some knowledge previously studied in subject 43483.

## OUTCOMES

### 2157 - M.D. in Research in Subject Didactics

- Students should apply acquired knowledge to solve problems in unfamiliar contexts within their field of study, including multidisciplinary scenarios.
- Students should be able to integrate knowledge and address the complexity of making informed judgments based on incomplete or limited information, including reflections on the social and ethical responsibilities associated with the application of their knowledge and judgments.
- Students should communicate conclusions and underlying knowledge clearly and unambiguously to both specialized and non-specialized audiences.
- Use appropriate bibliographical references that are relevant scientific background to the proposed research.
- Students should possess and understand foundational knowledge that enables original thinking and research in the field.
- Analyse and synthesise the main current research agendas in Specific Didactics.
- Conduct quality research in the scientific field of Specific Didactics using the methodologies, techniques and procedures of this discipline.
- Integrate ethical values and responsibility associated with research tasks into one's own research.
- Create spaces for research and learning with special attention to equity, emotional and values education, equal rights and opportunities between men and women, citizenship training and respect for human rights that facilitate life in society, decision-making and the construction of a sustainable future.
- Evaluate current research problems on teaching or learning in the fields of knowledge characteristic of Specific Didactics.
- Synthesise historical, epistemological and ontological aspects associated with the emergence and evolution of research in Specific Didactics.
- Evaluate the relevance of a research project, its quality and future projection, with scientific criteria appropriate to the international standards of the studied speciality.



- Synthesise relevant research problems on learning or teaching in the disciplines belonging to Specific Didactics.
- Search and synthesise information on research results in bibliographic, material, virtual, etc. repertoires useful to support a new research project.
- Critically analyse, from the point of view of research in Specific Didactics, the performance of teaching, good practice and guidance using quality indicators.
- Understand and apply specialised research procedures in Specific Didactics.
- Identify, analyse and evaluate national or international research publications in the field of Specific Didactics.
- Decide, with objective criteria, which methodological paradigm quantitative, qualitative or mixed best fits the objectives of your own research.
- Plantear preguntas de investigación pertinentes sobre un tema de investigación actual.
- Adequately analyse and evaluate the partial and final results of one's own research and contrast, refute or modify the first hypotheses.
- Choose an appropriate methodological framework to generate answers to research questions and master the use of the necessary methodological techniques.

## LEARNING OUTCOMES

The subject of Research in the Didactics of Elementary Mathematics must provide students with the highest level of specialisation in their research training. Students must complete the acquisition of the didactic and methodological knowledge obtained in other subjects with the most specialised knowledge necessary to successfully address the different tasks involved in carrying out a research project focused on elementary mathematical content or curricula.

By the end of the course, always referring to the research in the didactics of mathematics, students of this course are expected to have achieved the following results:

- Know specific elements of the main current lines of research in the didactics of elementary mathematics.
- Know the main theoretical frameworks specific to the teaching of elementary mathematics in detail.
- Use the knowledge acquired in this subject and other subjects to be able to critically analyse publications of research results.
- Present the results of their studies, literature analyses, etc., in a synthetic, complete and suitable way for an audience of researchers.
- Know elements of the history of mathematics that can be useful for aiding didactic research focused on elementary mathematics.



- Carry out epistemological reflections on school mathematics and students' conceptions.
- Raise issues that can serve as a basis for didactic research designs focused on elementary mathematical content and select appropriate theoretical frameworks and methodological tools to provide answers to such questions.

## DESCRIPTION OF CONTENTS

### 1. Research in the didactics of algebra at an early age

Overview of research on the teaching and learning of algebra at an early age.

2. Research on teaching and learning the algebra sign system at an early age.
3. Research on teaching and learning algebraic problem solving at an early age.
4. Research into the teaching and learning of functional thinking and pattern generalisation at an early age.

### 2. Research in the didactics of elementary geometry

1. Learning in 2D and 3D dynamic geometry software environments: instrumental genesis.
2. Cognitive effort in problem solving: levels of cognitive demand.
3. Research on learning mathematical demonstration.
4. Visualisation in the learning of mathematics. Research on the acquisition of visualisation skills and the flat representation of spatial objects.

### 3. Research in the didactics of elementary arithmetic

1. Teaching models relating to elementary arithmetic concepts:
  - The first number concepts and skills that develop before school age.
  - The development of natural number arithmetic, additive concepts and problems, and competencies that are common to primary school curricula in relation to computation (written, mental or estimated), and systematic errors in algorithms.

### 4. Research in teaching and learning of probability and statistics

1. Historical evolution of research in this field.
2. Current knowledge on research problems in different mathematical topics.

**WORKLOAD**

ACTIVITY	Hours	% To be attended
Theory classes	42,00	100
Development of group work	40,00	0
Development of individual work	40,00	0
Study and independent work	40,00	0
Readings supplementary material	13,00	0
<b>TOTAL</b>	<b>175,00</b>	

**TEACHING METHODOLOGY**

Various methodologies for teaching and student work will be applied, depending on the type of activity to be carried out. The following may be used:

- Lectures on the content given by teaching staff (usually in theory classes).
- Discussion between students under the observation of teaching staff, with or without their intervention (usually in seminars).
- Supervised or independent work, either individually or in small groups, to carry out projects, prepare materials, search for information, etc. (usually in the laboratory or as out-of-class activities)
- Supervised or independent individual study time (usually to prepare papers or assessment tests).
- Presentation of the work done in front of teaching staff and/or other students (usually in seminars).
- One-to-one meetings with the tutor to track the student's progress.

**EVALUATION**

Assessment will be based on the evaluation of evidence of learning, which may be collected by one or more of the following means:

- Regular tracking of students' progress in both theoretical classes and seminars, as well as in tutorials,
- Assessment of the required assignments.
- Individual and group participation in the activities carried out during theory classes and seminars (presentations of work, participation in discussions, etc.)
- Taking exams or other oral or written tests, designed to assess the students' level of proficiency in the subject competencies.





Each member of the teaching staff will be responsible for the assessment and grading of the part of the course that they have taught. For this purpose, the following shall be taken into account:

- The activities carried out by the students during the face-to-face class sessions (with a maximum value of 40%). These activities will only be counted when the student has attended at least 80% of face-to-face classes.
- Out-of-class assignments during the course or other assessment procedures that the teaching staff may determine (with a minimum value of 60%).

At the beginning of the course, each member of the teaching staff will report on the assessment procedure that they will apply and the distribution of percentages to be taken into account.

The final grade of the subject will be the weighted arithmetic average of the grades of the different members of the teaching staff. To pass the course, the grades from all members must be equal to or greater than 3.5 points out of 10 and the final grade of the subject must be equal to or greater than 5 points out of 10.

## REFERENCES

### Basic

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- Tema 3:
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