

**COURSE DATA****Data Subject**

Code	36591
Name	Espacios de Hilbert y series de Fourier
Cycle	Grade
ECTS Credits	6.0
Academic year	2022 - 2023

Study (s)

Degree	Center	Acad. year	Period
1928 - D.D. in Physics-Mathematics	Double Degree Program Physics and Mathematics	3	Second term

Subject-matter

Degree	Subject-matter	Character
1928 - D.D. in Physics-Mathematics	3 - Tercer Curso (Obligatorio)	Obligatory

Coordination

Name	Department
GALBIS VERDU, ANTONIO	15 - Mathematical Analysis

SUMMARY

The first part contains an introduction to the theory of Hilbert spaces, with special emphasis on the study of the orthogonal projection and the Fourier series with respect to an orthonormal system. Next we focus on the Fourier series with respect to the trigonometric system of periodic functions on the real line. The Dirichlet and Féjer nuclei are studied and some results are obtained regarding the convergence of the Fourier series. The course is completed with the study of the basic properties of the Fourier transform and Shannon's sampling theorem.

PREVIOUS KNOWLEDGE



Relationship to other subjects of the same degree

There are no specified enrollment restrictions with other subjects of the curriculum.

Other requirements

Mathematical analysis I F-M, Mathematical analysis II F-M

OUTCOMES

LEARNING OUTCOMES

To understand the foundations of Hilbert space theory and to know how to use the basic properties of classical spaces of functions and sequences. To learn the Fourier series development of a periodic function, as well as conditions that allow to recover a periodic function from its Fourier series. To know the basic properties of the Fourier transform, in particular the relationship with the convolution, the transform of a Gaussian and the inversion formula.

DESCRIPTION OF CONTENTS

1. Hilbert spaces

Pre-Hilbertian space. Cauchy-Schwarz and Minkowski inequalities. Associated norm. Identity of the parallelogram. Spaces ℓ_2 and $L_2(\omega)$

2. Optimal approximation

Distance from a point to a closed convex set. Orthogonal projection and orthogonal decomposition. Riesz representation theorem. Stampacchia and Lax-Milgram theorems.

3. Orthonormal systems

The trigonometric system in $L_2(T)$. Bessel inequality. Gram-Schmidt method. Maximal orthonormal systems. Parseval identity. Fourier coefficients. Riesz-Fisher theorem.

4. Fourier series in $L_1(T)$

Dirichlet and Fejér kernels. Convergence of the Césaro means in $C(T)$. Riemann-Lebesgue lemma. Completeness of the trigonometric system. Pointwise convergence of a Fourier series: Riemann's localization theorem and convergence to the mean of the jump when there are limits and lateral derivatives.

**5. Fourier transform in $L^1(\mathbb{R}^n)$**

The translation and modulation operators. Fourier transform and convolution. The transform of a Gaussian. Approximates of identity. Dirac delta and Heaviside function. Inversion formula. Shannon's sampling theorem: bandwidth of a signal and Nyquist frequency.

WORKLOAD

ACTIVITY	Hours	% To be attended
Theory classes	30,00	100
Classroom practices	22,50	100
Other activities	7,50	100
Development of individual work	10,00	0
Preparation of evaluation activities	40,00	0
Preparing lectures	10,00	0
Preparation of practical classes and problem	30,00	0
TOTAL	150,00	

TEACHING METHODOLOGY

The aim is to gradually introduce and develop the theoretical and practical contents of each topic and the right tools to solve problems.

In the practical sessions we will apply the concepts presented in the lectures to solve problems.

Questions and problems for study will be proposed. This study will be supervised and evaluated. In the practical sessions we will solve and correct exercises.

EVALUATION

Each student will have to demonstrate the knowledge of the basic concepts and the acquisition of the competences of the topic by means of the realization of theoretical-practical exams. Her ability to address the issues or solve the problems proposed by the teaching staff will also be valued.

The evaluation will be carried out by:

1. An exam at the end of the course, which will consist of a theoretical and a practical part that will each account for fifty percent of the grade, and the average will be made provided that each grade exceeds three points out of ten. A theoretical exam will measure both the acquisition of knowledge and the ability to write and rigor in the proofs, as well as the resolution of questions. In the practical exam the ability to solve problems and exercises will be evaluated.



2. Participation in the tasks or controls proposed by the teaching staff will be valued (10% of the grade), provided that the grade of the exams exceeds a minimum of four points.

3. Participation in the seminars (10% of the mark) will be valued, provided that the mark of the exams exceeds a minimum of four points.

REFERENCES

Basic

- J. Cerdá ; Introducció a l'Anàlisi Funcional. Publicacions i Edicions de la Universitat de Barcelona, 2005.
- K. Saxe; Beginning functional analysis. Undergraduate Texts in Mathematics. Springer-Verlag, New York, 2002.
- E.M. Stein, R. Shakarchi; Fourier Analysis: an Introduction, Princeton Lectures on Analysis, 2003.
- J. Duoandikoetxea; Lecciones sobre las series y las transformadas de Fourier.
<https://www.ugr.es/~acanada/docencia/matematicas/analisisdefourier/Duoandikoetxeafourier.pdf>
- G.B. Folland; Fourier analysis and its applications; Brooks/Cole Publishing, 1992.

Additional

- T.W. Körner, Fourier Analysis, Cambridge University Press, 1988.
- B. Cascales, J.M. Mira, J. Orihuela, M. Raja; Análisis funcional. Ediciones Electolibris, 2012.