COURSE DATA

Data Subject	
Code	36586
Name	Análisis matemático II F-M
Cycle	Grade
ECTS Credits	12.0
Academic year	2022 - 2023

Study (s)

Degree	Center	Acad. year	Period
1928 - D.D. in Physics-Mathematics	Double Degree Program Physics and Mathematics	2	Annual

Subject-matter

Degree	Subject-matter	Character
1928 - D.D. in Physics-Mathematics	2 - Segundo Curso (Obligatorio)	Obligatory

Coordination

Name	Department
MOLL CEBOLLA, JOSE SALVADOR	15 - Mathematical Analysis

SUMMARY

Mastering differential and integral calculus of functions of several real variables is one of the foundations of mathematics training. One of the objectives of the second degree course is the conceptual understanding and fluency in the use of some basic techniques in this matter.

The course is divided into two parts, each one is studied in a semester. In the first part we deal with Differential Calculus, which is developed for functions between finite dimensional euclidean spaces. The second part of the course is devoted to the study of the Lebesgue integral.

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PREVIOUS KNOWLEDGE

Relationship to other subjects of the same degree

There are no specified enrollment restrictions with other subjects of the curriculum.

Other requirements

Linear Algebra and Geometry I F-M, Mathematical Analysis I F-M

OUTCOMES

LEARNING OUTCOMES

- Calculate limits of functions of several variables and identify differentiable functions.
- Manage the partial derivatives using chain rule and implicit function theorem.
- Know the formulation of equations of mathematical physics through partial derivatives.
- Study local extrema and extrema subject to constraints of functions of several variables.
- Apply the inverse function theorem and Implicit function theorem to specific problems.
- Understand the concept of convergence of improper integrals and know the main convergence criteria.
- Know how to identify Lebesgue integrable functions.
- Apply the main theorems of convergence.
- Know the formulation of the theorems of Fubini, change of variables, and how to apply them for handling integration.
- Relate the concept of measure with that of integration.
- Solve problems involving integrals (lengths, areas, volumes and centers of gravity).

DESCRIPTION OF CONTENTS

1. Finite dimensional euclidean spaces

1.1 RN as a euclidean, normed and metric space.
Scalar product and euclidean norm in RN. Norm in RN. Distance in RN. Topological concepts. Distance from a point to a set. Distance between sets. Bounded sets.
1.2 Convergence in RN.
Convergent sequences. Sequence characterization of adherent and accumulation points
1.3 Compactness in RN.
Compact, relatively compact and bounded sets.

2. Continuous functions of several variables.

2.1 Functions between finite dimensional euclidean spaces. Limits. Definition of the limit of a function in an accumulation point of the domain. Sequence characterization of the limit. Projections. Coordinate functions. Arithmetic properties of the limits. Continuity in a point and in a set. Linear functions.
2.2 Complex functions. Continuity. Uniform branches of the argument.
2.3 Uniform continuity: Definition, Heine-Cantors theorem.

3. Differentiation

3.1 Directional derivatives and differential. Uniqueness of the differential. Relationship between continuity, differentiability and existence of directional derivatives. Jacobian matrix and gradient vector.
3.2 Complex differentiation. Cauchy-Riemann equations.
3.3 Chain rule in real and complex functions.
3.4 Mean value theorem and consequences.

4. Higher order derivatives.

4.1 Higher order partial derivatives. CK functions. A sufficient condition for differentiability. Cross derivatives theorems.
4.2 Taylors formula: Taylor expansion. Bounding Taylors remainder. Applications.
4.3 Local extrema. Critical points. Sufficient conditions for relative extrema. Hessian matrix.

5. Inverse and implicit function theorems

5.1 Non-null Jacobian functions.
5.2 Inverse function theorem in real and complex variables. Diffeormorphisms.
5.3 Implicit function theorem

6. Conditional extrema and Lagrange multipliers. Applications.

7. Lebesgue integrable functions

7.1 Null sets. Rectangles in RN. Measure of a rectangle. Null sets: Examples.
7.2 Step functions: characteristic function of a set. Step functions. Lebesgue integral of a step function. Properties.
7.3 Upper functions. Integral of upper functions.
7.4 Lebesgue integrable functions. Properties.
7.5 Characterization of Riemann integrable functions. Lebesgue-Vitali theorem. Improper Riemann integral.

8. Convergence theoremes

8.1 Monotone convergence theorem.
8.2 Dominated convergence theorem.
8.3 Fatous lemma

9. Fubinis theorem and applications

10. Measurable functions and Lebesgues measure

10.1 Measurable functions. Examples and properties.
10.2 Tonelli-Hobsons integrability criterion.
10.3 Measurable sets. Lebesgues measure in RN: Properties.
10.4 Open sets measurability.
10.5 An example of a non-measurable set.
10.6 Parametric integration.
10.7 Eulerian functions.

11. Integral transforms

11.1 Coordinate transforms.
11.2 Change of variables formula.

12. Lebesgues outer measure

12.1 Outer measure and regularity.
12.2 Egorov and Luzins theorems.
12.3 Characterization of measurable functions.
12.4 Vitalis covering theorem.

WORKLOAD

ACTIVITY	Hours	% To be attended
Theory classes	60,00	100
Classroom practices	45,00	100
Other activities	15,00	100
Development of group work	25,00	0
Study and independent work	50,00	0
Readings supplementary material	5,00	0
Preparation of evaluation activities	60,00	0
Preparing lectures	10,00	0
Preparation of practical classes and problem	30,00	0
TOTAL	**300,00**	

TEACHING METHODOLOGY

1. The aim is to gradually introduce and develop the theoretical and practical content of each topic and the right tools to solve problems.
2. In the practical sessions we will apply the concepts presented in lectures to solve problems.
3. We will propose questions and problems for study. This study will be supervised and evaluated. In the practical sessions we will solve and correct exercises.
4. We will use a symbolic computation software package that helps in the conceptual understanding and visualization. It will also serve as a testing method to provide intuitive knowledge

EVALUATION

Each student will be asked to demonstrate knowledge of basic concepts, skills and competences of the subject by means of theoretical and practical examinations. Also its capacity to address issues or resolve the problems posed by the teacher will be assessed.

Evaluation will be conducted by:

1. Written theory exams that will measure both the acquisition of knowledge and writing ability and rigor in proofs. Written practice exams will evaluate the ability to solve problems and exercises. There will be two exams throughout the course (middle and end of course). In each exam there will be a theoretical and a practical part which will contribute each fifty percent of the note provided that each note is greater than or equal to three out of ten. The note of each of the partial exams

must be greater or equal to four out of ten.
2. Participation on the tasks or controls proposed by the teacher will be evaluated (10%), provided that the obtained mark is above a minimum of four points.
3. Participation in the seminars will be evaluated (10%), provided that the obtained mark is above a minimum of four points.

REFERENCES

Basic

- Referencia b1: Mazón, J. M, Cálculo diferencial: Teoría y problemas, Publicacions de la Universitat de València, 2008.

  Referencia b2: Mazón, J. M, La integral de Lebesgue en RN. Teoría y problemas, Publicacions de la Universitat de València, 2017.

Additional

- Referencia c1: Apostol, T.M., Análisis Matemático, Editorial Reverté, 1977.

  Referencia c2: De Burgos, J. ,Cálculo infinitesimal de varias variables. Ed. McGraw-Hill, 1995.

  Referencia c3: Del Castillo, F. Análisis Matemático II. Ed. Alhambra, 1989.

  Referencia c4: Ortega, J. M. Introducció a l'Anàlisi Matemàtica. Manuals de la Universitat Autònoma de Barcelona, 1993.

  Referencia c5: Tao, T. Analysis II, Third Edition, Texts and Readings in Mathematics, Springer, 2016

  Referencia c6: Weir, A.J. Lebesgue Integration and Measure, Cambridge University Press, 1973.