

**COURSE DATA****Data Subject**

|                      |                                |
|----------------------|--------------------------------|
| <b>Code</b>          | 36136                          |
| <b>Name</b>          | Mathematics for dynamic models |
| <b>Cycle</b>         | Grade                          |
| <b>ECTS Credits</b>  | 6.0                            |
| <b>Academic year</b> | 2019 - 2020                    |

**Study (s)**

| <b>Degree</b>              | <b>Center</b>        | <b>Acad. year</b> | <b>Period</b> |
|----------------------------|----------------------|-------------------|---------------|
| 1316 - Degree in Economics | Faculty of Economics | 4                 | First term    |

**Subject-matter**

| <b>Degree</b>              | <b>Subject-matter</b>           | <b>Character</b> |
|----------------------------|---------------------------------|------------------|
| 1316 - Degree in Economics | 18 - Pathway: economic analysis | Optional         |

**Coordination**

| <b>Name</b>           | <b>Department</b>          |
|-----------------------|----------------------------|
| BOLOS LACAVE, VICENTE | 257 - Business Mathematics |

**SUMMARY**

"Mathematics for Dynamic Models" is an optional subject given in a semester of the fourth year of the Degree in Economics.

In this course, the concepts and basic techniques of resolution of difference and differential equations are developed, as well as the study of stability of fixed points (constant solutions).

In the first issue, basic concepts about discrete, continuous and stochastic dynamics are introduced. These concepts are necessary in order to modeling any kind of dynamical problem.

In the next issues, these concepts are extended and resolution techniques are developed, enlightened with some examples of known economical models.



## PREVIOUS KNOWLEDGE

### Relationship to other subjects of the same degree

There are no specified enrollment restrictions with other subjects of the curriculum.

### Other requirements

Prior knowledge corresponding to the following subjects are assumed:

- Mathematics I: Basic analytical concepts related to the differential and integral calculus.
- Mathematics II: Modelling of problems.

## OUTCOMES

### 1316 - Degree in Economics

- Have oral and written communication skills in the native language.
- Have decision-making skills and be able to apply knowledge to practice.
- Be able to learn autonomously.
- Apply the principles of economic analysis (rational decision) to the diagnosis and resolution of problems.
- Understand and apply the scientific method, which involves formulating hypotheses, deducing verifiable results and contrasting them with empirical and experimental evidence.

## LEARNING OUTCOMES

- Ability to recognize an economic problem from the observation of economic reality.
- Increment of the ability to use the logical and strategic thinking to address real situations of the economic world.
- Ability to select a theoretical framework for the development of analysis.

## DESCRIPTION OF CONTENTS

### 1. Introduction

1.1 Introduction to discrete dynamics. Difference equations (DEs). Introduction to continuous dynamics. Ordinary differential equations (ODEs). Introduction to stochastic dynamics.

1.2 Difference equations. Types of DEs. Difference operator. Solution of a DE. Initial value problem.

1.3 Ordinary differential equations. Types of ODEs. Solution of an ODE. Initial value problem.



1.4 Numerical methods of ODEs resolution. Discretization.

1.5 Examples. Discrete and continuous capitalization.

## **2. Difference equations**

2.1 First order autonomous DEs. Graphical resolution. Special solutions. Uniqueness of solution. Monotone DEs. First order implicit DEs.

2.2 Stability theory and fixed points. Attractors. Invariant sets. Stability of fixed points.

2.3 Higher order DEs.

2.4 Linear DEs. Resolution of homogeneous linear DEs. Resolution of complete linear DEs. Stability.

2.5 Examples. Economical models (french model of mortgage, supply-demand models, dynamics of market prices, market models with inventory,...).

## **3. Ordinary differential equations**

3.1 Special solutions. Existence and uniqueness of solutions of the Cauchy problem.

3.2 Resolution methods for first order ODEs. Equations of separable variables. Variable changes.

3.3 Linear ODEs. Resolution of first order linear ODEs. Linear ODEs with constant coefficients. Homogeneous linear ODEs. Non-homogeneous linear ODEs.

3.4 Examples. Economical models (models of inflation and unemployment, growth models of Domar and Solow, debt model of Domar, ...).

**WORKLOAD**

| ACTIVITY                                     | Hours         | % To be attended |
|--|---------------|------------------|
| Theory classes                               | 30,00         | 100              |
| Computer classroom practice                  | 30,00         | 100              |
| Development of individual work               | 15,00         | 0                |
| Study and independent work                   | 25,00         | 0                |
| Preparation of evaluation activities         | 15,00         | 0                |
| Preparing lectures                           | 15,00         | 0                |
| Preparation of practical classes and problem | 20,00         | 0                |
| <b>TOTAL</b>                                 | <b>150,00</b> |                  |

**TEACHING METHODOLOGY****- Lectures:**

The teacher will highlight the main aspects and those that are most difficult to understand, explain examples and guide the students through the available materials in the virtual classroom and the reference manuals. At the end of the lecture, the teacher will point out the materials needed for the next lecture, so the students can prepare it

**- Practical classes:**

The teacher will solve exercises and propose others that the student must solve either on the board or in the next class.

**EVALUATION**

The student must submit some exercises designed by the teacher throughout the course, which will be evaluated together on 5 points. At the end of the semester, a written exam of 5 points will be made. The final grade will result from the sum of the notes of the exercises and the written exam.

On second call, a written exam of 5 points will be made, keeping the note of the exercises. However, the student can renounce the note of the exercises and so, the written exam will be of 10 points.

In subsequent calls, a written exam of 10 points will be made.



## REFERENCES

### Basic

- G. Gandolfo. Economic Dynamics. Ed. Springer (1997).
- H. Lomelí, B. Rumbos. Métodos Dinámicos en Economía. Ed. Thomson (2003).
- R. Shone. Economic Dynamics. Cambridge University Press (1997).
- K. Sydsaeter, P.J. Hammond. Matemáticas para el Análisis Económico. Ed. Prentice-Hall (1996).
- D.G. Zill. Ecuaciones diferenciales con Aplicaciones. Grupo Ed. Iberoamericano (1986).
- C. Fernández, F.J. Vázquez, J.M. Vegas. Ecuaciones Diferenciales y en Diferencias. Sistemas Dinámicos. Thomson-Paraninfo (2004).

### Additional

- A.C. Chiang. Métodos Fundamentales de Economía Matemática. McGraw-Hill (1987).
- A. de la Fuente. Mathematical Methods and Models for Economists. Cambridge University Press (2000).
- G. Gandolfo. Métodos y Modelos Matemáticos de la Dinámica Económica. Biblioteca Tecnos de Ciencias Económicas (1976).
- M.W. Klein. Mathematical Methods for Economics. Adisson-Wesley (1998).
- S.N. Nefcti. An Introduction to the Mathematics of Financial Derivatives. Academic Press (2000).
- J.T. Sandefur. Discrete Dynamical Systems. Oxford (1990).

## ADDENDUM COVID-19

**This addendum will only be activated if the health situation requires so and with the prior agreement of the Governing Council**

In case of not being able to take classes or exams in person, they will be carried out on-line through the Aula Virtual.