

**COURSE DATA****Data Subject**

<b>Code</b>	34175
<b>Name</b>	Group theory
<b>Cycle</b>	Grade
<b>ECTS Credits</b>	6.0
<b>Academic year</b>	2022 - 2023

**Study (s)**

<b>Degree</b>	<b>Center</b>	<b>Acad. year</b>	<b>Period</b>
1107 - Degree in Mathematics	Faculty of Mathematics	4	First term

**Subject-matter**

<b>Degree</b>	<b>Subject-matter</b>	<b>Character</b>
1107 - Degree in Mathematics	16 - Seminar on Algebra	Optional

**Coordination**

<b>Name</b>	<b>Department</b>
BALLESTER BOLINCHES, ADOLFO	363 - Mathematics
ESTEBAN ROMERO, RAMON	363 - Mathematics

**SUMMARY**

English

Whether one wants to study groups because of its applications or to do research in the theory, the concept of action is implicit in the nature of the groups, initially as permutation groups but also as transformations or actions on objects and structures of various kinds.

It is thus necessary to study actions on groups and its application to the construction of the semidirect product. It is related and of great interest the Schur-Zassenhaus theorem.

The idea of solubility appears at the origin of the theory of groups linked to the solubility by radicals of algebraic equations. Its influence affects the arithmetic structure and its normal structure of groups. Burnside's theorem on the solubility of groups whose order is divisible only by two primes and Hall's theorems are key examples of this.



## PREVIOUS KNOWLEDGE

### Relationship to other subjects of the same degree

There are no specified enrollment restrictions with other subjects of the curriculum.

### Other requirements

Knowledge of the course on Algebraic Structures

## OUTCOMES

### 1107 - Degree in Mathematics

- Capacity for analysis and synthesis.
- Solve problems that require the use of mathematical tools.
- Learn autonomously.
- Possess and understand the mathematical knowledge.
- Apply the knowledge in the professional world.
- Expressing mathematically in a rigorous and clear manner.
- Knowing the time and the historical context in which occurred the great contributions of women and men in the development of mathematics.
- Visualize and interpret the solutions obtained.

## LEARNING OUTCOMES

Students should develop those learning skills necessary to undertake further studies with autonomy.

Students should know how to use search tools for library resources.

Students should learn how to do presentations and expose their work in public.

Students should be able to be familiar with topics of current interest of recent research in group theory.

Students should acquire techniques to study abstract groups from the knowledge of families of relevant subgroups.

Students should recognise solubility of finite groups from their arithmetical structure.

Students should express in an algorithmic way the theoretical proofs to solve concrete problems.



## DESCRIPTION OF CONTENTS

### 1. Revision and preliminaries

Especificación de contenidos de la unidad

We review previous knowledge about permutation groups, soluble groups, and Sylow theory. The concepts of commutator, minimal normal subgroup and maximal subgroup are introduced and the basic properties proved. Sylow-type subgroups are defined and their elemental properties are showed. Sylow-type subgroups are defined and their elementary properties are proved.

### 2. Nilpotent groups. Fitting and Frattini subgroups

The properties of the  $p$ -groups, the center of a nontrivial  $p$ -group is nontrivial, every proper subgroup is contained in its normaliser itself serve as a basis for introducing nilpotent groups by the central series. The product of nilpotent normal subgroups of a finite group is a nilpotent normal subgroup.

We study the main properties of the Frattini subgroup and Fitting subgroup in the universe of all finite groups.

### 3. Primitive groups. Galois Theorem

We introduce the concept of primitive group, and prove theorem of Galois about primitive groups.

### 4. Semidirect product. Schur-Zassenhaus. Hall's theorem.

We introduce the concept of semidirect product, so necessary for the examples, and theorem of Schur-Zassenhaus.

With the techniques already developed, we are able to prove the fundamental theorem of Hall on finite soluble groups.

**WORKLOAD**

ACTIVITY	Hours	% To be attended
Theory classes	37,50	100
Classroom practices	15,00	100
Other activities	7,50	100
Study and independent work	16,50	0
Readings supplementary material	8,00	0
Preparation of evaluation activities	16,50	0
Preparing lectures	24,80	0
Preparation of practical classes and problem	24,70	0
Resolution of case studies	8,50	0
<b>TOTAL</b>	<b>159,00</b>	

**TEACHING METHODOLOGY**

The subject has 30 hours of theory class distributed in two sessions of 1 hour per week and 15 hours of problem classes distributed in sessions of two hours and that will be given at the rate of a maximum of one session per week. There are also 5 seminar sessions of 1.5 hours that will take place during 5 weeks of the semester. Attendance at both theory classes and problem classes is strongly recommended. In the theory classes we will give the necessary and most important tools for understanding and solving problems. In the problem classes, the assimilation and better understanding of the concepts developed in the theoretical classes will be deepened by solving problems and exercises. This work will be carried out through the explanations made by the teacher on the blackboard and the active participation of the students in the discussion of the different arguments used in solving the problems. This subject will also offer resources through the Aula Virtual. In it we will incorporate the statements of the lists of problems and additional material that can complement the classes of theory and problems.

**EVALUATION**

The mark obtained in the exam will count 80 % of the final grade. The seminar will note the 10 % and 10 % participation.



To pass you must obtain a minimum grade of 3,2 out of 8 on the exam.

In the second call, the assessment system will be the same. **The scores of the seminars and the participation cannot be recovered in the second call.**

## REFERENCES

### Basic

- Referencia b1: Isaacs, I. M. Finite Group Theory, AMS 2008
- Referencia b2: Kurzweil, H., Stelmacher, B. The Theory of Finite Groups, Springer-Verlag, 2004
- Referencia b3: Robinson, Derek J.S. A course in the theory of groups, Springer-Verlag, 1980
- Referencia b4: Rose J.S., A Course on Groups Theory, Cambridge U.P., 1978

### Additional

- Referencia c1: Doerk, K., Hawkes, T.O., Finite Soluble Groups, Walter de Gruyter, 1992.
- Referencia c2: Huppert, B., Endlichen Gruppen I, Springer-Verlag, 1967
- Referencia c3: Gorenstein, D., Finite Groups, Chelsea, 1980