



## COURSE DATA

## Data Subject

Code	34174
Name	Harmonic analysis
Cycle	Grade
ECTS Credits	6.0
Academic year	2023 - 2024

## Study (s)

Degree	Center	Acad. year	Period
1107 - Degree in Mathematics	Faculty of Mathematics	4	Second term

## Subject-matter

Degree	Subject-matter	Character
1107 - Degree in Mathematics	15 - Seminar on Mathematical analysis	Optional

## Coordination

Name	Department
BELTRAN PORTALES, DAVID	15 - Mathematical Analysis

## SUMMARY

Harmonic Analysis deals with the representation of functions as series or integrals of simpler ones.

For periodic functions on  $\mathbb{R}$  this leads to the representation of the function as a series of sines and cosines, called Fourier series.

In the case of functions defined on  $\mathbb{R}$  or  $\mathbb{R}^n$ , this leads us to study the Fourier transform.

Some summability methods of convergence of Fourier series and their analogues for Fourier transforms are studied. The notion of convolution between functions in both  $\mathbb{T}$  to  $\mathbb{R}$  is considered and used to approximate and regularize functions as well as to give some summability and convergence results. The study of Fourier series and Fourier transform of square integrable functions, specially the



Plancherel theorem, are also key points in the development of the subject.

## PREVIOUS KNOWLEDGE

### Relationship to other subjects of the same degree

There are no specified enrollment restrictions with other subjects of the curriculum.

### Other requirements

Linear Algebra and Geometry I and Mathematical Analysis I, II, III, IV.

## OUTCOMES

### 1107 - Degree in Mathematics

- Capacity for analysis and synthesis.
- Capacity for criticism.
- Solve problems that require the use of mathematical tools.
- Ability to work in teams.
- Learn autonomously.
- Possess and understand the mathematical knowledge.
- Expressing mathematically in a rigorous and clear manner.
- Capacity of abstraction and modeling.
- Knowing the time and the historical context in which occurred the great contributions of women and men in the development of mathematics.
- Visualize and interpret the solutions obtained.

## LEARNING OUTCOMES

To Know and to apply sufficient conditions for recovering a periodic function from its Fourier series, and their immediate application to compute the sum of some convergent series of real or complex numbers.



To know and to apply sufficient conditions for recovering a function from its Fourier transform.

To learn how to apply Fourier series and Fourier transforms to solve certain types of differential equations.

## DESCRIPTION OF CONTENTS

### 1. Introduction to harmonic analysis

Partial differential equations: harmonic oscillator, wave equation and heat equation and their relationship with Fourier series.

### 2. Fourier series

Fourier Series. Convergence criteria for Fourier series. Summability of Fourier series

### 3. Fourier transform

Convolution and regularization of functions. Fourier transform on  $L^1$ . Fourier transform on  $L^2$ . Plancherel Theorem

### 4. Applications

## WORKLOAD

ACTIVITY	Hours	% To be attended
Theory classes	37,50	100
Classroom practices	15,00	100
Other activities	7,50	100
Development of group work	10,00	0
Development of individual work	10,00	0
Study and independent work	25,00	0
Readings supplementary material	5,00	0
Preparation of evaluation activities	20,00	0
Preparation of practical classes and problem	5,00	0
Resolution of case studies	15,00	0
<b>TOTAL</b>	<b>150,00</b>	



## TEACHING METHODOLOGY

- a. The aim is to gradually introduce and develop the theoretical and practical contents of each topic and the right tools to solve problems.
- b. In the practical sessions we will apply the concepts presented in the lectures to solve problems.
- c. Questions and problems for study will be proposed. This study will be supervised and evaluated. In the practical sessions we will solve and correct exercises.
- d. We will use a symbolic computation software package that helps in the conceptual understanding and visualization. It will also serve as a testing method to provide intuitive knowledge.

## EVALUATION

Evaluation will be conducted through:

- 1) A written theory exam will measure both the acquisition of knowledge and writing ability and rigor in proofs. A written practice exam will evaluate the ability to solve problems and exercises.
- 2) Participation in the tasks or controls proposed by the teacher will be evaluated (10 %).
- 3) Participation in the seminars will be evaluated (10 %).



## REFERENCES

### Basic

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Referencia b1: Stein, Shakarchi; Fourier Analysis: an Introduction, Princeton Lectures on Analysis, Zaanen.

Referencia b2: Ducandikoetxea; Lecciones sobre las series y las transformadas de Fourier, Apuntes de Managua, 2003.

Referencia b3: Dym, McKean; Fourier Series and Integrals, Academic Press, 1973.

Referencia b4: Zaanen, A.C.; Continuity, integrations and Fourier theory; Springer-Verlag, 1989.

### Additional

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Referencia c1: Katznelson, an introduction to harmonic analysis. Dover Publications, 1976.

Referencia c2: Körner, Fourier analysis, Cambridge University Press, 1988.