COURSE DATA

Data Subject	
Code	34164
Name	Topology
Cycle	Grade
ECTS Credits	12.0
Academic year	2022 - 2023

Study (s)

Degree	Center	Acad. year	Period
1107 - Degree in Mathematics	Faculty of Mathematics	2	Annual
1928 - Double Degree Program Physics-Mathematics	Double Degree Program Physics and Mathematics	2	Annual

Subject-matter

Degree	Subject-matter	Character
1107 - Degree in Mathematics	9 - Topology and differential geometry	Obligatory
1928 - Double Degree Program Physics-Mathematics	2 - Segundo Curso (Obligatorio)	Obligatory

Coordination

Name	Department
NUÑO BALLESTEROS, JUAN JOSE	363 - Mathematics
OSET SINHA, RAUL ADRIAN	363 - Mathematics

SUMMARY

The overall objective of this course is to introduce students to the basics of topology. Most of the course is devoted to general topology, which provides basic language for understanding other subjects such as geometry or analysis. We will also introduce at the end of the course certain concepts less instrumental and more typical of other variants of the topology, such as geometric topology and algebraic topology.

Topology is the branch of mathematics devoted to the study of those properties of geometric shapes that do not depend on quantities and are invariant under continuous transformations. This study is based on the concept of proximity, and allows us to establish an axiomatic approach to the concepts of neighborhood, open set, closed set, continuity, and so on, using as main tool the language of set theory.

Based on the previous experience of the student with the topology of the real line, we introduce first metric spaces, prior to the further abstraction of topological space. Next we will study ways of getting new examples of topological spaces by the construction of subspaces, products and quotients. To finish the general part, we introduce the most important topological properties (connection and compactness) and metric properties (completeness).

Finally we dedicate the last part of the course to the classification of compact surfaces, and a brief introduction to the fundamental group. These are concepts more applicable to geometric topology (or to low dimensional topology) and algebraic topology and which require further development of the geometric intuition of the students.

The contents of this course are metric spaces, topological spaces, countability properties, separation, convergence and continuity, subspaces and products of topological spaces, compactness and completeness, connection and introduction to the fundamental group, quotient topological spaces and the description of compact surfaces.

PREVIOUS KNOWLEDGE

Relationship to other subjects of the same degree

There are no specified enrollment restrictions with other subjects of the curriculum.

Other requirements

It is desirable that the student has completed the first year core subjects, especially Basic Mathematics and Analysis I.

COMPETENCES (RD 1393/2007) // LEARNING OUTCOMES (RD 822/2021)

1107 - Degree in Mathematics

- Capacity for analysis and synthesis.
- Solve problems that require the use of mathematical tools.
- Ability to work in teams.
- Learn autonomously.
- Possess and understand the mathematical knowledge.
- Expressing mathematically in a rigorous and clear manner.
- Reason logically and identify errors in the procedures.
- Capacity of abstraction and modeling.
- Knowing the time and the historical context in which occurred the great contributions of women and men in the development of mathematics.
- Visualize and interpret the solutions obtained.

LEARNING OUTCOMES (RD 1393/2007) // NO CONTENT (RD 822/2021)

• Manage with ease the basic topological concepts in Euclidean spaces.

• Use sequences to characterize the basic topological concepts in metric spaces.

• Recognize equivalents metrics, and examples of non-metrizable topological spaces.

• Analyze the continuity of applications, both from a local and global standpoint.

• Recognize connection properties and compactness in simple topological spaces.

• Build examples of topological spaces using the notions of subspaces, products or quotients.

• Recognize topologically compact surfaces and their classification.

DESCRIPTION OF CONTENTS

1. Metric spaces

Definition and examples of metric spaces.
Balls. Bounded metric spaces.
Open subsets and properties.
Neighborhoods. Closed subsets.

2. Topological spaces

Definition and examples of topological spaces.
Closed subsets. Neighborhoods.
Countability Axioms and Hausdorff topological spaces.
Equivalent metrics.

3. Particular points

Adherency points and related concepts.
Boundary points. Interior points.
Characterization by sequences.

4. Continuity

Continuity of a map at a point.
Global continuity of a map.
Uniform continuity and isometries.

5. Subspaces

Relative topology.
Relative closure, interior and boundary.
Continuity and subspaces.

6. Connection

Connection.
Connected subsapaces of R.
Other properties of the connection.
Path connection.

7. Products

Product topology.
Closure, interior and boundary of a product.
Continuity and products.

8. Compactness

Definition and Examples.
Compact subspaces. Characterization of compact subspaces of R and Rn.
Relationship with continuous maps.
Sequentially compact spaces.

9. Completeness

Complete metric spaces.
Some theorems on complete spaces.

10. Quotients

Definition and basic properties.
Relationship with subspaces and products.
The Hausdorff property in quotiens.

Definition and basic properties.
Relationship with subspaces and products.
The Hausdorff property in quotiens.

11. Fundamental group

Definition of fundamental group.
Continuous maps and fundamental group.
The fundamental group of the circle.
The Brouwer fixed point theorem in dimensión 2.

12. Classificacion of surfaces

Definition and examples of surfaces.
Triangulation of compact surfaces.
Orientable and nonorientable surfaces.
Classification of compact surfaces.
The Euler characteristic.

WORKLOAD

ACTIVITY	Hours	% To be attended
Theory classes	60,00	100
Classroom practices	45,00	100
Other activities	15,00	100
Development of individual work	15,00	0
Preparation of evaluation activities	60,00	0
Preparing lectures	60,00	0
Preparation of practical classes and problem	30,00	0
TOTAL	**285,00**	

TEACHING METHODOLOGY

The theoretical part will be developed in lectures where the lecturer will introduce gradually the mathematical content and method. On each U.T., besides the theoretical knowledge, the lecturer will include quite a number of examples and will solve exercises specific to that part. At the end of each U.T. the lecturer will provide lists of exercises to be solved by the students.

The more practical part will developed in smaller groups where students will practice working in stable groups of three or four students under the supervision of the teacher. Each group will hand in writing the answers of the exercises to be marked by the teacher.

Both theoretical and practical classes will make use of tools for visualization of geometric objects.

Finally, there will be regular seminars in which the students will solve doubts and will discuss with the teacher those aspects of the subject they deem appropriate. In addition, different activities to be undertaken by students under teacher supervision shall be proposed.

EVALUATION

The assessment of learning knowledge and skills achieved by students will be made continuously throughout the course, and consists of the following items:

1. **Tests**: Two written tests of a theoretical-practical nature one at the end of each semester, with a weight of 60% of the final grade.

2. **Practice**: assessment of participation in practice sessions and the written presentation of the results of these sessions. This will have a weigh of 30% of the final grade.

3. **Tutorials and Seminars**: assessment of participation in tutorial sessions and seminars and carrying out the proposed activities. The weight will be 10% of the final grade.

Comments:

• Block 1 requires a minimum mark of 4/10 in each test in order to average with blocks 2 and 3.

• Marks in blocks 2 and 3 are kept if there is a resit exam of the course in the same academic year.

REFERENCES

Basic

- F. Mascaró, J. Monterde, J.J. Nuño i R. Sivera, Introducció a la topologia. Universitat de València (1997).
- W.S. Massey, Introducción a la topología algebraica. Reverté (1982).

Additional

- M.A. Armstrong, Topología Básica. Reverté (1987).

- J.R. Munkres, Topologia (2ª Edición) Prentice-Hall (2002).