



## COURSE DATA

Data Subject	
<b>Code</b>	44076
<b>Name</b>	Fundamentals of advanced mathematics
<b>Cycle</b>	Master's degree
<b>ECTS Credits</b>	6.0
<b>Academic year</b>	2024 - 2025

## Study (s)

Degree	Center	Acad. Period year
2183 - Master's Degree in Mathematical Research	Faculty of Mathematics	1 First term

## Subject-matter

Degree	Subject-matter	Character
2183 - Master's Degree in Mathematical Research	7 - Fundamentals of advanced mathematics	Obligatory

## Coordination

Name	Department
FALCO BENAVENT, FRANCISCO JAVIER	15 - Mathematical Analysis
RUEDA SEGADO, MARIA PILAR	15 - Mathematical Analysis

## SUMMARY

The course focuses on Measure Theory and its applications. After developing the notions of measure on a sigma-algebra and the construction processes of measures from outer measures, specific cases such as Lebesgue measure in  $\mathbb{R}^n$  and Borel-Stieltjes measures on intervals will be constructed.

The student's familiar concepts of measurable and integrable functions will be reviewed, along with classical theorems (monotone convergence, dominated convergence of Lebesgue, Fubini's theorem) in the general context of abstract measures. The key result of the course will be the Radon-Nikodym theorem and some of its applications.



## PREVIOUS KNOWLEDGE

### Relationship to other subjects of the same degree

There are no specified enrollment restrictions with other subjects of the curriculum.

### Other requirements

The student should be familiar with the basic tools of integration in one and several variables.

## COMPETENCES (RD 1393/2007) // LEARNING OUTCOMES (RD 822/2021)

### 2183 - Master's Degree in Mathematical Research

- Students should apply acquired knowledge to solve problems in unfamiliar contexts within their field of study, including multidisciplinary scenarios.
- Students should be able to integrate knowledge and address the complexity of making informed judgments based on incomplete or limited information, including reflections on the social and ethical responsibilities associated with the application of their knowledge and judgments.
- Que los estudiantes comprendan los conceptos y las demostraciones rigurosas de teoremas fundamentales de alguna de las áreas específicas de las Matemáticas.
- Que los estudiantes sean capaces de aplicar los resultados y técnicas aprendidas para la resolución de problemas complejos de alguna de las áreas de las Matemáticas, en contextos académicos o profesionales.
- Que los estudiantes tengan capacidad para elaborar y desarrollar razonamientos lógico-matemáticos e identificar errores en razonamientos incorrectos.
- Que los estudiantes sean capaces de construir, interpretar, analizar y validar modelos matemáticos avanzados que simulen situaciones reales.
- Que los estudiantes sean capaces de comprender de manera autónoma artículos de investigación o innovación en alguna de las áreas de las Matemáticas.

## LEARNING OUTCOMES (RD 1393/2007) // NO CONTENT (RD 822/2021)

- Rigorously justify and construct the Lebesgue measure in multiple dimensions, as well as the Lebesgue-Stieltjes measure, starting from the general formalization of the notions of sigma-algebra of sets and measures defined on abstract sigma-algebras.
- Study measurable and integrable functions with respect to general measures in the general context.



- Ultimately demonstrate the important Radon-Nikodym theorem for representing absolutely continuous measures with respect to a given measure.

## DESCRIPTION OF CONTENTS

### 1. Abstract measure

1. Outer measures. Extension of measures. Fourier-Stieltjes measures. Measurable and non-measurable sets.

### 2. Measurable functions and integrable functions

1. Measurable functions. Some types of convergence. Integrable functions.

### 3. Product measure and Fubini's theorem

1. Product measure. Fubini's theorem. Applications.

### 4. The Radon-Nikodym theorem

1. Complex and real measures. The Radon-Nikodym theorem. Applications.

## WORKLOAD

ACTIVITY	Hours	% To be attended
Theory classes	60,00	100
Development of individual work	30,00	0
Study and independent work	30,00	0
Readings supplementary material	30,00	0
<b>TOTAL</b>	<b>150,00</b>	

## TEACHING METHODOLOGY

Combination of lectures and presentations by students on selected topics. In face-to-face classes, the theoretical content of each topic and the appropriate tools for problem-solving will be introduced and developed progressively. A series of results, questions, and problems will be proposed for study, applying the concepts covered in the theoretical classes. Students will be required to present their solutions.



## EVALUATION

La asignatura se evaluará mediante la presentación de problemas y cuestiones relativos a la materia propuestos de manera individualizada, o bien mediante la exposición en pizarra de una parte del curso por parte del alumno. También se propondrán trabajos realizados individualmente o en grupo y su correspondiente exposición en clase.

## REFERENCES

### Basic

- Bartle, R. *The elements of integration and Lebesgue measure*. Wiley classics Library. Edition 1995.
- M. de Guzmán; B Rubio. *Integración, Teoría y Técnicas*. Ed. Alhambra, 1979.
- M. Valdivia Ureña, *Análisis Matemático V*. UNED. Edición 2002.
- Mukherjea, A.; Pothoven, K. *Real and functional analysis. Part A. Real analysis*. Second edition. Mathematical Concepts and Methods in Science and Engineering, 27. Plenum Press, New York, 1984.

### Additional

- George, C. *Exercises et problems of integration*. Gauthier-Villars, Paris, 1980.
- W. Rudin, *Analisis real y complejo*. Mac Graw-Hill, 1988.